

REVIEW

CHARACTERISTIC FEATURES OF CONVECTIVE HEAT TRANSFER AT A LIQUID-SOLID BODY PHASE INTERFACE

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Introduction. The process of heat transfer of a liquid during ice formation presents a rather complex problem, since freezing means not only the transition of a substance from one state to another with different thermophysical properties, but also the liberation of the latent heat of melting. Problems of this type fall into a wide class of problems known as the Stephan problem [1]. In its original statement, i.e., on the assumption that there is no convection in a liquid and that the thermophysical properties are constant, one finds the law of phase interface motion in time by solving heat conduction equations with nonlinear boundary conditions. The basic methods of solving such problems have been classified and are most fully presented in [2].

Allowance for such factors, very frequently occurring in real situations, as multidimensionality, boundedness in space, liquid supercooling below crystallization temperature, and certain aspects of convective heat transfer at the phase interface greatly complicates the statement of the corresponding problems, the solution of which requires special mathematical techniques [3-7], approximate numerical methods and supercomputers [8], or similarity theory methods with a large number of definitive criteria [9]. The insuperable mathematical difficulties that sometimes arise compel investigators to resort to a traditional technique, i.e., to use the methods of classical heat conduction theory and to take into account the above-mentioned factors in boundary conditions determined experimentally.

All of the above-enumerated features of the process require a special detailed examination; however, the amount of research carried out by the present time makes it possible to analyze only some of them.

The aim of the present review is to summarize research efforts over the past years devoted to the determination of the laws governing convective heat transfer at the "liquid-solid body" phase interface.

Effect of the Formed Solid Phase Shape on Heat Transfer of the Surface Immersed in an Infinite Flow (Exterior Problem). The first attempts to analyze the process of heat transfer in the period of transition from the liquid to the solid state under forced convection conditions were undertaken in [10-12], where the authors limited their discussion primarily to a qualitative description of the process.

A detailed investigation of the coupling between the phase interface surface shape and heat transfer in the case of flow past a frosted plate was reported in [13, 14]. The theoretical analysis is restricted to problems of laminar flow with a relatively simple geometry of the phase interface surface. The one-dimensional heat transfer model developed is based on the assumption of the presence of a thin ice layer; therefore when constructing the solution the heat transfer coefficient is assumed to be independent of the shape of the ice and is defined in the same way as for a plate with no ice ("clean" plate). Analysis of the process with the aid of a two-dimensional model shows that a parabolic ice layer is formed on the plate, with the parabola tip being displaced forward from the leading edge of the plate to a certain nominal distance. In this case, the heat transfer coefficient is calculated with the use of the laminar boundary layer theory relations. Comparison of the predicted results with experimental data shows that application of a simple one-dimensional approximation is more expedient in the majority of practical cases [13].

The transition from a laminar to turbulent flow regime on the ice surface differs substantially from the process of transition on a clean plate [14]. Figure 1 schematically shows the regimes of flow and the shape acquired by the surface of ice in each regime.

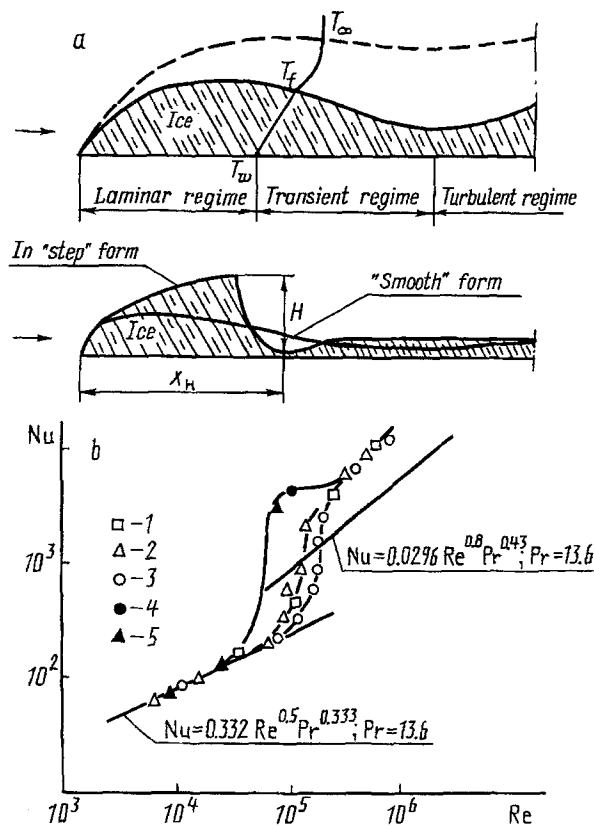


Fig. 1. Ice surface shape (a) and heat transfer (b) in transient and turbulent flow regimes [14]: solid curve — heat transfer from a "clean" surface; 1-3) heat transfer for "smooth" ice; 4, 5) heat transfer for ice in the form of steps.

It turns out that depending on the behavior of the ice thickness there exist two different forms of ice surface in the transient flow regime: the "step-like" and "smooth." In the case of the "smooth" form, the ice thickness gradually decays in the entire transient regime, whereas in the case of the "step-like" form it undergoes transformation in a sudden jump at a certain point where an ice thickness maximum is observed. In the course of time this point can be displaced toward the leading edge of the plate, accounted for by the local melting of ice. A dimensionless parameter has been found which uniquely determines the shape of the surface in the transient regime: when $Re_x^{0.13} H/x_H > 0.15$ the "step-like" form of the ice surface develops, and when $Re_x^{0.13} H/x_H < 0.15$ the "smooth" surface is formed. The critical Re_x number at which the flow regime alters lies within $7 \cdot 10^4 - 1 \cdot 10^5$ for a "smooth" transition as against $1 \cdot 10^5$ for a "clean" plate. Experimental investigation of heat transfer has shown that with the "step-like" form of the ice surface the transition to a turbulent mode is accompanied by a substantial (by a factor of 1.5-2.5) rise in the heat transfer rate at the ice-water interface as compared with a "clean" plate (Fig. 1b). This trend of investigation was further developed in [15], where the transition from the laminar to turbulent mode of flow was investigated in the presence of a rather thick layer of ice. It is shown that on compliance with certain regime conditions on the ice-water surface, one can observe an instability in the existence of the plane phase interface. Wavy ice appears which represents a system of periodic crests and troughs. The authors defined the region for the existence of such a form of the ice surface by the following inequality: $\Theta_c > 12$.

The thickening of ice leads to stable growth in the heat transfer rate. Thus, as compared with a clean surface the heat transfer flux from wavy ice is higher by 30-60%. The reason for this is an additional agitation of the flow by each wave formed on the ice surface.

In [16] heat transfer with the formation of ice on the outer surface of a cylinder in a cross flow is studied. Within the scope of the experiments carried out it was found that the thickness of the nonfrozen layer increases monotonically from $\varphi = 0$ to $\varphi \sim 120^\circ$ and that it is virtually constant within the range $120^\circ \leq \varphi \leq 180^\circ$. These results allowed one to represent the nonfrozen layer profile as: the form of the ice from the forward stagnation point to $\varphi \sim$

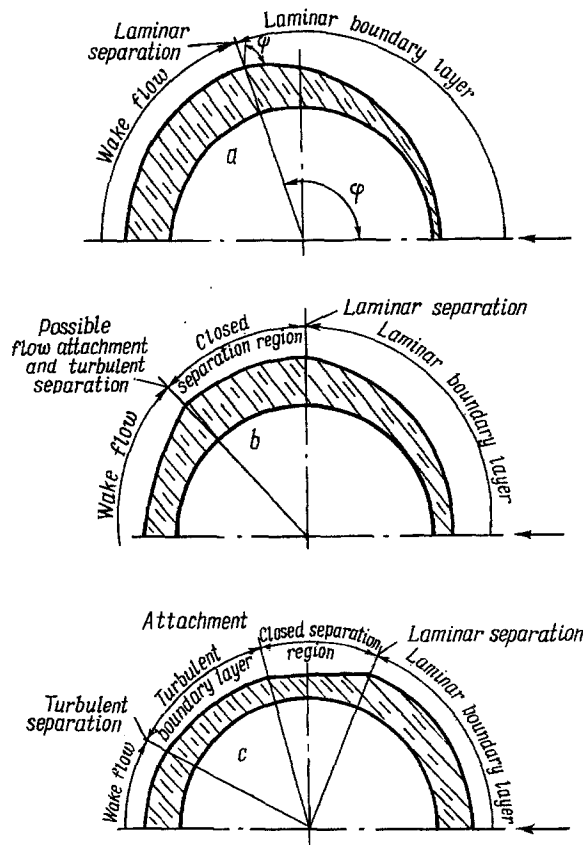


Fig. 2. Ice layer profiles on the outer surface of a cylinder in a cross flow at different Re_d numbers [17]: a) $Re_d = 2.08 \cdot 10^3$, $\Theta_c = 10.9$; b) $1.88 \cdot 10^4$ and 75.8 ; c) $5.82 \cdot 10^4$ and 63.2 .

120° is an eccentric circle whose center is displaced from the cylinder center in the flow direction by the distance b , and the form of the rear portion (from $\varphi \sim 120^\circ$ to $\varphi \sim 180^\circ$) is a concentric circle around the cylinder. Since the local heat transfer coefficient has its maximum at the forward stagnation point and falls down to $\varphi \sim 120^\circ$ and the changes in the heat transfer rate in the rear region are insignificant for small Re_d numbers, such a distribution of ice thickness around the perimeter corresponds to local heat transfer around the cylinder. Such a conclusion allowed the authors to employ correlations of the local heat transfer coefficient distribution around a clean cylinder for constructing an analytical model of the process in the following form:

$$\alpha(\varphi) = 1,03 \left\{ 1 - 2,41 \left(\frac{\varphi}{\pi} \right) + 0,74 \left(\frac{\varphi}{\pi} \right)^2 \right\} \frac{\lambda_l}{d} \left(\frac{r_p}{r} \right)^{1/2} Re_d^{1/2} Pr^{1/3},$$

where $0 \leq \varphi \leq 2/3\pi$.

This trend of investigation was extended in [17], where a steady-state process of two-dimensional ice buildup around an isothermally cooled circular cylinder in a cross flow was investigated experimentally within the ranges of Re_d numbers from $2.3 \cdot 10^2$ to $8.6 \cdot 10^4$ and of the thermal parameter of cooling Θ_c from 6.3 to 75.6 . The dependence of the ice thickness on the angular coordinate φ was determined by visualizing and taking photographs of stationary ice profiles. The local and averaged heat transfer coefficients on the ice-water phase interface surface were determined by solving the Laplace equation for the ice layer. Comparing the behavior of these coefficients with the change in the local heat transfer coefficient, the authors draw a hypothetical conclusion regarding the possible flow regimes (Figs. 2 and 3). At the smallest Re_d number ($Re_d = 2.08 \cdot 10^3$) in the region bounded by the value $0 \leq \varphi \leq 120^\circ$, a laminar boundary layer is formed. When $\varphi \sim 120^\circ$, this layer separates and the wake flow is realized. The authors suggest to perform a comparison of the location of the separation point for the frosted and "clean" cylinders using the angle between the local normal to the ice surface and the incident flow direction ψ . A comparison based on this parameter shows that the position of the laminar boundary layer separation points is preserved and is governed by the angle

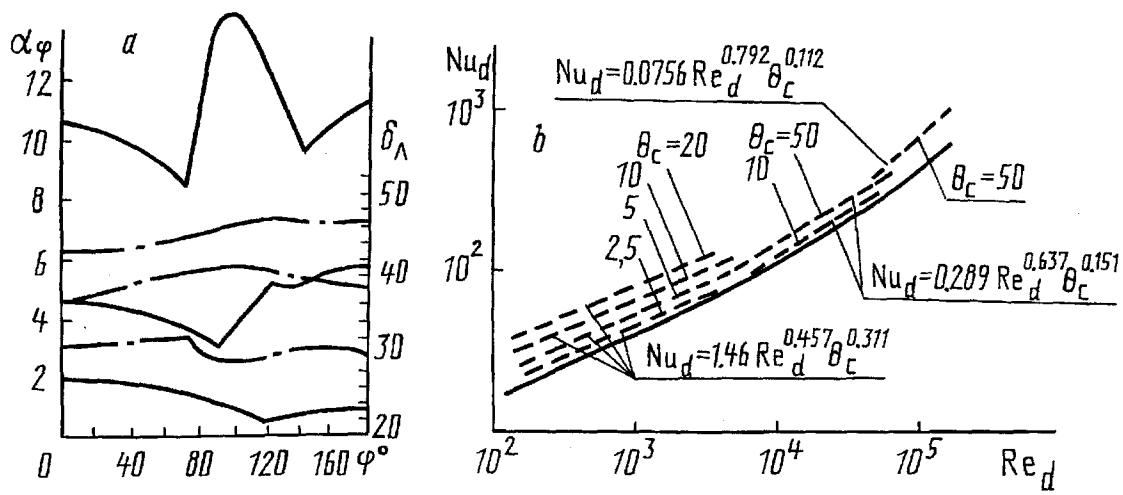


Fig. 3. Variation in the ice layer thickness and in the local heat transfer coefficient around the cylinder perimeter [17]: dashed-dotted curves, ice profile δ_i ; solid curves, heat transfer coefficient α_φ and ice thickness effect on the mean rate of heat transfer to the cylinder [17] (solid curve, heat transfer from the "clean" cylinder). α_φ , W/(m²·K); δ_i , mm.

$\psi \approx 80^\circ$. At higher Re_d numbers ($Re_d = 5.82 \cdot 10^4$) there is a laminar boundary layer in the region with $0 \leq \varphi \leq 70^\circ$. Laminar separation originates at the point $\varphi = 70^\circ$, and up to $\varphi = 100^\circ$ there is a closed separation region. At the point $\varphi = 100^\circ$ the attachment of the turbulent boundary layer takes place, which develops over the ice surface up to the point $\psi \approx 140^\circ$, where flow separation takes place again. The form of the ice profile at the intermediate Reynolds number ($Re_d = 1.88 \cdot 10^4$) shows that this number is close to the smallest one at which a turbulent boundary layer originates. The existence of the closed separation region and the marked decrease in the critical Reynolds number (by about an order of magnitude), at which transition from the laminar to the turbulent boundary layer takes place, is, in the authors' opinion, an essential feature of the hydrodynamic pattern of the process considered.

Variation of the mean Nusselt number Nu_d with the Reynolds number is shown in Fig. 3, where the results on heat transfer of a "clean" cylinder are given for comparison.

Empirical heat transfer relations are given in the following form:

$$Nu_d = 1,46 Re_d^{0,457} \theta_c^{0,311} \text{ when } 2 < \theta_c < 20 \text{ and } Re_d < 5 \cdot 10^6,$$

$$Nu_d = 0,289 Re_d^{0,637} \theta_c^{0,151} \text{ when } 6 < \theta_c < 50 \text{ and } 76,5 \cdot 10^3 < Re_d < 5 \cdot 10^4,$$

$$Nu_d = 0,0756 Re_d^{0,797} \theta_c^{0,112} \text{ when } 40 < \theta_c < 65 \text{ and } Re_d > 5 \cdot 10^4.$$

Effect of the Formed Solid Phase Shape on Heat Transfer in Fluid Flow Through Channels (Interior Problem). The effect of solidification on heat transfer during fluid motion through a channel formed by two parallel plates is considered in [6, 18, 19].

Work [18] is concerned with a theoretical analysis of the given problem for fluids with a low Pr number in the case of developed turbulent motion. Numerical and analytical solution of the system of heat balance equations revealed a strong effect of the temperature ratio $\lambda_i(T_f - T_w)/\lambda_l(T_0 - T_f)$ on the solid phase thickness.

The aim of work [19] was to investigate the possibility for applying the analytical method suggested in [20] to liquid flow between parallel plates. The feasibility of such an approach is shown in principle, but determination of the ice profile becomes complicated and requires cumbersome computations.

In [6] theoretical consideration is given to laminar fluid flow in the presence of a cooled starting length. Results are obtained on the position of the liquid-solid body phase interface profile and the hydraulic drag; the effect of the thermal conductivity ratio λ_i/λ_l on local heat transfer at $Pr = 13.2$ is determined.

Experimental results on heat transfer characteristics along the ice-liquid phase interface for water flow between two horizontal parallel plates are presented in [21]. It is found that to correlate experimental data it is advisable to employ the number Re_h and the relative thermal head Θ_c .

Visual observations revealed that the stable existence of two types of ice profile was possible: "transient" and "smooth." They are similar to those presented in Fig. 1. The criterion which nonambiguously determines the existence of a particular profile is the following relation:

$$\frac{Re_h}{\Theta_c^{0.741}} < 10^4.$$

The transient type of ice profile owes its origin to the hydrodynamic instability of the laminar boundary layer, which goes over into a turbulent layer at a certain point of the channel on variation in external conditions. Flow separation leads to an increase in the local coefficient of heat transfer from the water to the ice surface. Let us determine the coordinates of the onset of separation point formation

$$\frac{x_{tr}}{h} = 6,95 \cdot 10^5 Re_h^{-1,1} \Theta_c^{-0,123}$$

and of its steady-state position

$$\left(\frac{x_{tr}}{h} \right)_{st} = 2,46 \cdot 10^5 Re_h^{-1,02} \Theta_c^{-0,113}.$$

The results of investigation into local heat transfer show that with the "transient" type of the ice profile the heat transfer coefficient has a pronounced maximum at the separation point. In the case of the "smooth" type of ice formation the value of the heat transfer coefficient remains almost constant over the entire plate length, except for the natural increase over the starting length. Mean heat transfer with the "transient" type is defined as

$$Nu_h = 0,875 Re_h^{0,382} B_F^{0,637} \left(\frac{x_{tr}}{h} \right)_{st}^{-0,127},$$

and with the "smooth" type as

$$Nu_h = 7,37 \cdot 10^{-2} (Re_h \cdot B_F)^{0,624},$$

where

$$B_F = \frac{\lambda_i}{\lambda_l} \frac{T_f - T_w}{T_\infty - T_f}$$

is the freezing parameter.

The relations obtained describe the experimental data within 25%. They are applicable in a rather wide range of Re_h values (from $3.8 \cdot 10^3$ to $3.2 \cdot 10^4$).

Among the first works concerned with the refinement of the laws of convective heat transfer of a freezing liquid moving in a tube is the investigation conducted by Gilpin [22], who visualized the process in a vertical tube. He found that in contrast to theoretical predictions the ice profile forms not a uniformly converging channel, but has a complex structure with cyclic variations of the cross section along the tube length. In this case, the liquid flow in a tube was close to the transient one.

The natural desire of the author to expand the range of investigations was realized in [23], where the maximum Re number attained $1.4 \cdot 10^4$. The experiments carried out revealed that the transient ice layer development in a tube depended on the type of flow (laminar or turbulent) which existed in the tube prior to the ice formation. Figure 4 demonstrates the growth of ice with time on the inner tube surface. Up to the time $\tau \simeq 2$ h the formation of ice followed the theory (a monotonic thickening along the tube length). However, at the time instant $\tau \simeq 4$ h there was an abrupt expansion of the channel at the outlet from the test section. This expansion point was displaced slowly upstream. Below this abrupt change in the cross section the ice melted. When the transition point moved rather far

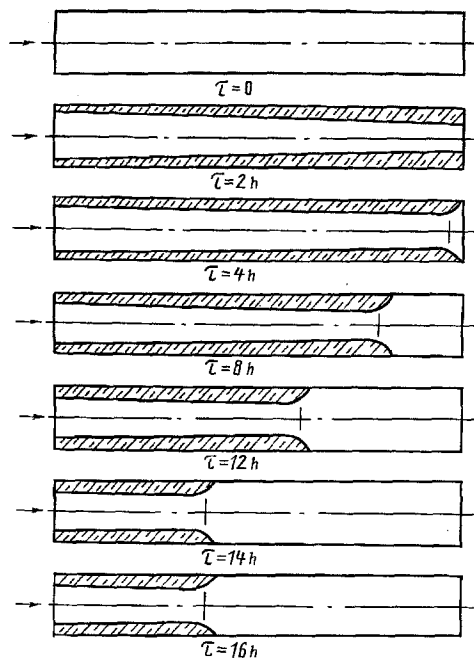


Fig. 4. Ice layer development in a tube in the case of an initially laminar flow [23]: $Re = 3025$; $\Theta = 2.6$.

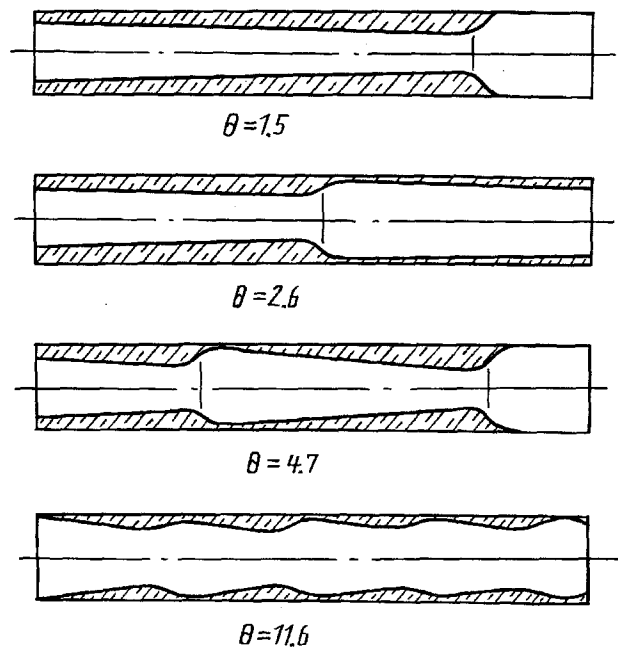


Fig. 5. Different stationary forms of ice depending on the thermal head Θ ; $Re = 3025$ [23].

from the tube outlet, new formation of ice became possible near it under certain conditions (see Fig. 4, $\tau = 14$ h). With the passage of time, the above-described sequence of events led to the formation of a number of ice "corrugations" separated by "clean" portions.

An increase in the relative thermal head increases the number of expansion and contraction cycles in the channel (see Fig. 5). Moreover, it is noted that the system of ice "corrugations" is typical only for $\Theta > 1$, whereas with $\Theta < 1$ a uniformly contracting channel is formed.

The instability of the initial ice profile form is explained by the presence of a large perturbation in the flow, viz., by a sudden expansion at the tube end leading to the incipience of vigorous turbulence. Due to ice melting at the separation point, the perturbation propagates upstream up to the final steady state determined by the local heat balance.

With the initial flow being turbulent, the ice profile structure develops in another fashion. At $Re \approx 9100$ ice corrugations and separation points are formed along the entire tube length. In the author's opinion, each wave originates due to the macrononuniformities available on the wall. The final structure resembles the form of the ice existing at small Re numbers, attested to by the absence of the influence of the initial Re number on the final stationary ice profile. By analyzing the experimental data obtained and comparing them with a "smoothly frosted" tube it was shown that the abrupt expansion-contraction of the flow in the complex system of ice "corrugations" made the major contribution to the considerable (by an order of magnitude or higher) increase in the hydraulic drag of a partially frosted tube. The absence of results on heat transfer prevented a conclusion about the quantitative measure of the change in heat transfer rate. However, there is no question that during the transition from "smooth" frosting to the formation of a system of ice confusor-diffusor-type channels, a kind of heat transfer enhancement takes place by analogy with the results presented in [15, 21].

Work [7] is a continuation of a series of investigations concerned with the study of heat transfer of a freezing liquid in a tube. The investigations showed that the establishment of the steady-state flow in a partially frozen tube was preceded by the appearance of decaying fluctuations consisting in periodic repetitions of the "melting-solidification" process. Such a phenomenon leads to fluctuations of hydraulic drag with an amplitude of about 120 mm of water and a period of $\approx 25-35'$. It has been possible to find that the range of their existence is very narrow

In [24] the problem of heat transfer in laminar liquid flow in a vertical tube is considered and solved numerically. The temperature of the inner tube wall is below the liquid solidification point. In the numerical method used the interphase boundary was converted to a surface with a constant radial coordinate. This considerably simplified the algorithm of calculation. Comparison between the predicted and experimental data showed that good agreement was obtained only when $B_F \leq 2$. At large values of this parameter the flow develops turbulent perturbation due to the laminar boundary layer instability and wavy surface formation. In [25] a correlation between the phase interface front position and the dynamics of the wall temperature variation was found experimentally for a convectively cooled tube in the interior of which crystallization of a liquid took place. The measured data were compared with the results of numerical calculations within the framework of two different models. It was shown that using simple relations obtained in a quasi-steady approximation, it was possible to determine the current position of the phase interface with satisfactory accuracy.

Work [26] considers the results of numerical and experimental investigation into the process of heat transfer under more complex conditions of liquid cross flow past a bundle of tubes whose surface is covered with a layer of ice. On the basis of the developed physical model of uniform frosting, a system of differential equations describing heat transfer in a coiled heat exchanger was integrated numerically. Heat transfer at the phase interface was described by the correlation suggested by A. ukauskas for a "clean" bundle. Comparison of the results obtained with experimental data shows that there are regions of good agreement (with the discrepancy not exceeding 15%) and unsatisfactory agreement (with the discrepancy being above 80%). The results of the analysis agree with the data of visual observations. Thus, for the built-up ice form close to the uniform one (both around the perimeter of a separate pipe and over heat exchanger model layers), good agreement between experimental and calculated data is typical.

At lowered Re numbers one observes the preference in the growth of ice on the lower layers of tubes, which ultimately leads to the formation of a "bypass" liquid flow over the upper layer of tubes with greatly deteriorated thermohydraulic characteristics. Such a form of ice compromises the assumptions adopted when constructing the computational model of the process and is responsible for a considerable discrepancy between experimental and predicted data in the region of low Re numbers and thermal heads Θ .

The limit for the existence of uniform frosting of a bundle of tubes and correspondingly the region of the applicability of the computational model are determined by the onset of the natural convection effect and are characterized by the relation $Gr/Re^2 < 0.04$. This value has been determined by comparing predicted and experimental data on heat transfer and hydraulic drag and has been confirmed by the results of visual observations of the process of ice formation.

Conclusion. Investigations carried out to date are predominantly devoted to the study of the specific features of convective heat transfer at the liquid-solid body phase interface during liquid flow in channels and past surfaces of initially simple (prior to ice formation) geometric form. The authors note some fundamental differences of the process studied as compared with flow in "clean" channels. These are: the earlier origination of the turbulent mode of flow (according to [14, 17], the critical Re number decreases virtually by an order of magnitude) and the considerable increase in the heat transfer rate.

Investigations of the exterior problem (a plane single plate immersed in a flow, a single cylinder in an infinite cross flow) show that the formation of ice leads to a considerable enhancement of heat transfer in the region of transient and turbulent modes of flow [14, 16, 17]. Experimental data on heat transfer have been generalized in the form of correlations where the Reynolds number and, as a rule, the relative ice thickness are included as governing parameters. A similar approach is also realized in the analysis of the results of investigation of heat transfer characteristics along the "ice-liquid" phase interface during water flow between two horizontal parallel plates [21]. This transfer of a bundle of tubes in a cross flow with a layer of ice formed on their surfaces was investigated in [26], where the authors determined the region of applicability of the well-known heat transfer and hydraulic drag relations. Outside the boundaries of this region there exists a "bypass" flow characterized by greatly deteriorated thermohydraulic characteristics.

NOTATION

x_H , distance from the plate leading edge to the point with maximum heat transfer coefficient; H , maximum ice thickness in the case of "step-like" transition; T_F , T_w , T_l , T_c , T_∞ , temperature of the phase change, wall, liquid, coolant, and the main stream, respectively; φ , angular coordinate; ψ , angle between local normal to ice surface and main stream direction; b , magnitude of frosted ice layer eccentricity; α , heat transfer coefficient on ice surface; r_p , radius of "clean" cylinder; r , radius of forward portion of frosted ice layer; d , diameter of "clean" cylinder; δ_i , ice thickness; λ_i , λ_l thermal conductivity of ice and liquid; $\Theta = (T_F - T_w) / (T_\infty - T_F)$, $\Theta = (T_F - T_c) / (T_l - T_F)$, relative thermal head; $Re = \omega_\infty x / \nu$, $Re = \omega_\infty d / \nu$, $Re = \omega_y d / \nu$, Reynolds number where ω_∞ is the main stream velocity, ω_y is the velocity in "narrow" section of "clean" bundle, x is the linear coordinate, ν is the kinematic viscosity; $Pr = c_p \mu / \lambda$, Prandtl number where c_p is the isobaric heat capacity, μ is the kinematic viscosity; $Nu = \alpha d / \lambda$, $Nu_h = \alpha 2h / \lambda$, Nusselt number where h is the distance between "clean" plates; $Gr = g \beta d^3 (T_l - T_w) / \nu^2$, Grashof number where g is the free fall acceleration, β is the coefficient of volumetric expansion; tr, st, "transient" and stationary forms of ice.

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